into a number of independent families of equal rank. These are based on the study of skeletal structures, known facts of anatomy, and phylogenetic relationships. She arranges the families, according to the lines of descent demonstrated in the present paper, as follows:—

Zaphrentoidean Families: Zaphrentidæ, Amphiastræidæ, Turbinolidæ, Stylinidæ, Oculinidæ, Pocilloporidæ, Madreporidæ, Poritidæ.

Cyathophylloidean Families: Cyathophyllidæ, Astræidæ, Fungidæ, Eupsammidæ.

V. "On the Calibration of the Capillary Electrometer." By George J. Burch, M.A. Communicated by Professor B. Price, F.R.S. Received June 24, 1895.

In my papers* "On a Method of Determining the value of Rapid Variations of a Difference of Potential," and "On the Time-Relations of the Capillary Electrometer," I showed that the photographic record of an excursion of the capillary electrometer indicates electromotive changes not only qualitatively but quantitatively, even when they last too short a time for the movements of the meniscus to be completed.

The movement of the mercury commences simultaneously with the communication to it of the E.M.F., and ceases the instant it is withdrawn, the velocity of the meniscus at any instant being proportional to the difference between the P.D. of the source and the P.D. of the charge in the electrometer. For the sake of brevity, this difference will be referred to as the Acting P.D.

I showed that if an excursion is recorded in the form of a curve, of which the abscissæ are proportional to the times and the ordinates to the acting P.D.'s, the acting P.D. at any instant is given immediately by the tangent to the curve at that point; and the method of analysis set forth in my papers was based on the determination of the tangent or its equivalent.

Since then the process has been applied to several hundred photographs, most of which were taken during the research on the electrical phenomena of muscle and nerve in which I have assisted Professor J. Burdon Sanderson.†

In the apparatus finally adopted the sensitive plate is fixed to one end of a balanced pendulum by which it is carried with uniform velocity past a vertical slit. The image of the capillary is thrown

* 'Roy. Soc. Proc.,' vol. 48, p. 89; and 'Phil. Trans.,' vol. 183, A, pp. 81—105.

† 'Physiol. Soc. Proc.,' June 24, 1893, in 'Journ. of Physiol.,' vol. 16, p. 319, and vol. 18, p. 171.

upon this slit, so that the movements of the meniscus are recorded in a polar curve in which time is measured by the angular displacement, and the position of the mercury by the radius vector.

In such a curve, the total indicated E.M.F. is made up of two parts, viz.:—

(A) A part indicated by the distance the meniscus has moved from its zero position. This corresponds to the scale readings of the instrument, as used for measuring a steady difference of potential.

(B) A part indicated by the velocity with which the meniscus is moving. This is proportional to the polar sub-normal to the curve, which can be easily measured.

The algebraic sum of these two parts represents the total P.D. of the source at that instant. By repeating the process for a number of points on the curve, a derived curve can be drawn showing the actual variations of the E.M.F. during the passage of the plate.

Considered as a practical means of measuring electromotive changes of short duration, the method set forth in my paper is open to one grave objection.* It assumes that the time relations of the apparatus employed correspond to the formula

$$r = ae^{-c\theta}.$$

To expect the maker of the electrometer to guarantee this would greatly enhance the cost of an instrument necessarily fragile, and it would be equally fatal to the general adoption of the method if the experimenter had to go through a tedious series of measurements with each new capillary.

The process of calibration which I am about to describe is a general one, applicable to all kinds of dead-beat instruments of which the excursions can be recorded in polar curves, and by an obvious modification applicable also to curves with rectangular co-ordinates. It takes into account all "errors" except overshooting, and this, in the case of the capillary electrometer, can be got rid of by inserting a sufficient external resistance. A point of great practical importance is that no special care is needed in photographing the three "Normal Excursions" required for the purpose.

The principle of the method, so far as it relates to the calibration of the sub-normal, which is the only difficult part, is as follows:—

Assuming that the resistance in circuit is such as to render the electrometer perfectly dead-beat, we may extend the law of the time-relations of the movement to include all forms of capillary, thus:—

The ratio of the velocity of the meniscus to the difference between the E.M.F. of the source, and that of the charge contained in the electrometer, is constant for any given part of the capillary, but may

* 'Phil. Trans.,' vol. 183, A, p. 101.

vary for different points along its length. It is required to determine the amount of this variation.

Two normal excursions are photographed. In one, the zero is below the field of view, and the movement is directed upwards. In the other, the zero is raised above the field of view, by a suitable alteration of the pressure bulbs, and the connections with the potentiometer are reversed, so that the movement of the meniscus is downwards. The exact limits and relative position of the two excursions are immaterial so long as the capillary itself has not been shifted. The object is to obtain two curves in opposite directions, running right across the plate.

Let p be a point on the capillary, the position of which is determined by its distance from the reference-circle, or upper limit of the photograph, in each case.

Using V_p to represent the acting P.D., indicated by the velocity of the meniscus at the point p, we may write

$$-V_p = -x \text{ for the upward excursion,}$$

and similarly

$$+V'_p = +x' \text{ for the downward excursion.}$$

Let $-a =$ the P.D. necessary to bring the meniscus from p to a point q, above p. Then by the known law of the capillary electrometer $+a$ is the P.D. necessary to bring the meniscus from q to p. And the acting P.D. at q is

$$-V_q = -x+a \text{ for the upward excursion,}$$

and

$$+V'_q = +x'+a \text{ for the downward excursion.}$$

Subtracting, we have

$$V_q+V'_q = x+x' = V_p+V'_p.$$

That is to say, in any pair of oppositely directed excursions which overlap, the algebraic difference between the acting P.D. of the upward movement and the acting P.D. of the downward movement at the same level is constant for all points common to both curves.

But the velocity of the meniscus at any point is measured by the polar subnormal to the curve, whence it follows that "*The algebraic difference between the polar subnormals to corresponding points upon two oppositely directed excursions is constant if the time-relations of the instrument agree with the formula* $r = ae^{-c\theta}$." That is to say, representing the subnormals to the two excursions by the letters N and N′ respectively,

$$k(N_p+N'_p) = x+x' = k(N_q+N'_q).$$

With a capillary of different form, this relation will no longer hold. The velocity of the meniscus may be different, for the same acting P.D., at the points p and q. But the value of the subnormal at q is at once given by the formula

$$\frac{N_p+N'_p}{N_q+N'_q} \cdot k \cdot N_q = x'.$$

The constant k is most conveniently determined by measuring the subnormal at the point p on a third photograph in which a normal excursion of known value starts from a zero point within the field of view.

Practice of the Method.

It may be convenient to describe briefly the entire process of calibrating an electrometer. In order to ensure that the tube shall always occupy the same position, the image of the tip of the capillary is adjusted to a mark on the back of the exposing shutter. It is of course not perfectly in focus, but is sufficiently sharp for the purpose. This mark serves for all experiments with the same capillary.

Calibration of the Scale Readings.

A millimetre scale on glass is fixed to the focussing screen so that its zero coincides with the upper limit of the slit. The slit and the cylindrical lens are then removed, and the image of the meniscus adjusted to zero on the scale.

The E.M.F.'s necessary to bring the mercury down to 5, 10, 15, &c., mm. on the glass scale are then measured with a potentiometer, care being taken to short-circuit between each observation lest the zero should alter. With very delicate instruments, or if the pressure tubes are not air-tight, it is sometimes easier to measure the length of the excursion produced by a small constant P.D. in various parts of the tube, and obtain the value of the scale-readings by calculation.

Calibration of the Subnormal.

Two normal excursions in opposite directions are photographed. Each must be large enough to traverse the full length of the slit during the passage of the plate, but not much more, otherwise the curves will be too steep for accurate measurement.

A non-inductive, non-polarisable resistance of at least 10,000 ohms must be included in the circuit. The subnormals to these curves are then measured at points 5, 10, 15, &c., mms. from the reference circle.

As an illustration I give the results obtained with an electrometer which had been in use for a long time. The tube had been cali-

brated by the old method, but had become "sticky" at its best part, and the new process was therefore applied to the portion below the obstructions and extending beyond the region of equal sensitiveness.

Distance from the reference-circle, in mms.	Subnormals.			Value per centimetre of the subnormal.	
	Upward excursion $-N$.	Downward excursion $+N$.	$N + N'$.	Relative $\frac{N_p + N'_p}{N_q + N'_q}$.	In volts.
0	Tube sticky. Value per cm., as found by previous calibration, constant throughout this part.			—	0·01333
20	−13·0	+42·0	55·0	1·000	0·01333
25	−16·5	+38·5	55·0	1·000	0·01333
30	−21·5	+33·0	54·5	1·009	0·01345
35	−25·0	+29·0	54·0	1·018	0·01357
40	−30·5	+21·5	52·0	1·058	0·01411
45	−34·5	+16·0	50·0	1·100	0·01466
50	−37·0	+ 9·5	46·5	1·183	0·01577

The fourth column shows the initial velocity of the excursion caused by the same difference of potential, for seven equidistant positions of the meniscus. But the exact value of this difference of potential is unknown, because the zero-positions of the excursions are not recorded. A third photograph is therefore taken in order to determine the value in volts of a centimetre of the subnormal at some one of the positions in the table. To do this, the potentiometer is set accurately to give some known P.D. The zero point of the meniscus is brought about 30 mm. below the reference circle, and the excursion is directed upwards. The subnormal to the resulting curve is measured at a point 25 mm. from the reference-circle, *i.e.*, in the middle of the available space. It would be difficult to make the excursion start *exactly* from the 25 mm. mark, and besides it is easier to place the tangent-line accurately against the curve at a point some distance from its origin.* The value of the acting P.D. at the point measured is found by subtracting the P.D. corresponding to the distance through which the meniscus has risen, from the P.D. indicated by the potentiometer.

In the case quoted, the acting P.D., at a point 25 mm. from the reference circle, was 0·02 volt, and the subnormal to the curve at this point was 15 cm.

* *Note added November* 19, 1895.—The shape of this curve at its origin shows whether the instrument is dead-beat or not. If the velocity of the meniscus increases after the excursion has begun, the capillary should be rejected.—G. J. B.

The numbers in the fifth column are obtained from those in the fourth column by dividing each of them by 55. The last column needs no explanation. The time spent in taking these photographs, and another set nearer the tip of the tube, was about one hour, and the calibration of the scale readings and measurement of the subnormals of both sets, was effected in an hour and a half. The entire calibration covered 95 mm. of the projected image of the capillary, but the upper part was completely spoilt by the stickiness of the tube. One advantage of this process is that it enables such defects to be detected by inspection. Each mechanical obstruction leaves a mark on both curves, but most evidently on that which has the smaller subnormal. The effect is equivalent to a loss of time in the excursion. The meniscus is retarded in the act of passing the point, but immediately afterwards resumes its normal velocity. It would be quite possible, if it were worth while, to measure the "work done" in passing a sticky place.

It should be observed that if the value of the scale-readings is known, the calibration of the subnormal can be effected by means of a single excursion of known P.D., provided that the zero-point is visible on the photograph. For the acting P.D. at any point on the curve can be found by subtracting from the E.M.F. given by the potentiometer, the P.D. corresponding to the rise of the meniscus; and the ratio $\frac{\text{acting P.D.}}{\text{subnormal}}$ for a series of equidistant points, gives the calibration-curve of the capillary. This was the method I employed in 1891, but it requires much greater care to get concordant results, and I realised that until some simpler process could be found, few people would care to use this method of determining the value of rapid changes of potential difference with the capillary electrometer.

Note.—Professor Einthoven, in a paper published in 1894,* recommends that in selecting an instrument, care should be taken that it is of equal sensitiveness throughout the working portion of the tube, and assumes that such a test is a sufficient criterion of its fitness for use.

This I have not found to be the case. A constant subnormal is always associated with a slight increase of sensitiveness towards the tip of the capillary.

More recently† he has put forward a statement having a very important bearing on this problem. He now finds that in some electrometers the ratio of the velocity of the meniscus to the acting P.D. is not constant, but diminishes as the mercury approaches its position of rest. In such cases he proposes to construct a curve of

* 'Archiv für die Ges. Physiologie,' vol. 56, p. 528.

† 'Archiv für die Ges. Physiologie,' vol. 60, pp. 91—100, and 101—123.

constants representing the variations of this ratio for different values of the acting P.D. It is hardly necessary to point out that this is quite a different thing from my "calibration curve," which relates to the variation of the time-relations at *different parts of the same capillary*.

With regard to Professor Einthoven's observations, I can only say that I have not met with the phenomenon save in "sticky" electrometers; and in such cases minute irregularities are plainly visible on the curves, when the plates are driven fast enough. But the rate I employ for physical investigations, namely, from 1,000 to 1,500 mm. per second, is so many times greater than that used in the examples he gives, viz., 20 to 25 mm., that I am able to detect what would otherwise be invisible. His instruments would appear to be much less rapid and less sensitive than most of mine, but he uses a higher magnifying power for the projection. The actual distance traversed by the meniscus itself in making the excursion, of which the analysis was given in my paper,* was fifteen times as great as that traversed by the meniscus of his electrometers in the eight excursions adduced in support of his new theory. But although those excursions are smaller and slower, the curves are much steeper than I should deem advisable, especially as the length measured, according to his plan, is the reciprocal of the ratio to be determined.

That there must have been either irregularities in the curves, or considerable error in the measurements, is manifest when, instead of taking the average values as he has done, the eight curves are reduced to a common scale, and plotted on the same sheet of paper. One of them especially, No. 205 A, differs from all the rest in form.

By using the asymptote to the normal curve as a line of reference, Professor Einthoven introduces a source of error. The slightest stickiness may cause the meniscus to stop short of the asymptote, or an insignificant amount of overshooting may carry it beyond, and a difference which would appear in the fifth place of decimals in estimating an E.M.F. becomes of crucial importance in calculating the time-relations of the movement. It is for this reason that I use the upper limit of the photograph as a reference circle, and reckon the displacement of the meniscus from its zero-position.

I have, since reading Professor Einthoven's paper, again calibrated, by both the methods described in this communication, the electrometer in use in the Physiological Laboratory at Oxford. Both calibrations agree, proving that the instrument shows no trace of the peculiarity mentioned by him. Should I detect it in any future electrometers, I will not fail to communicate the fact to the Society.

* 'Philosophical Transactions,' vol. 183, A, p. 95.